

$$\underline{I} \ 1) \ 60 - 20 : 2 = 60 - 10 = 50$$

$$2) \ \frac{1}{4} \cdot \frac{120}{30} = 30$$

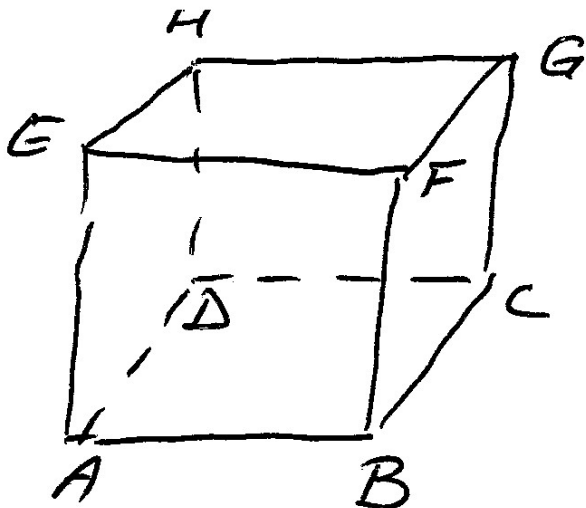
3) cel mai mic nr natural din  
 $I = [10, 20]$  este 10

$$4) \text{ aria } \triangle MPQ = l^2 = 25 \text{ cm}^2$$

$$5) \ m(\angle BC, \widehat{D\Delta'}) = 90^\circ$$

$$6) \ \text{grâu } 20 \text{ Kg}$$

II) 1)



$$2) a = \left( \frac{3}{4} - \frac{1}{2} \right) \cdot \frac{1}{12} = \frac{1}{4} \cdot \frac{12}{1} = 3$$

$$b = 3 \cdot \left( \frac{2}{3} - \frac{1}{2} + \frac{1}{6} \right) = 3 \cdot \frac{2}{6} = 1$$

$$m_{\text{air}} = \frac{a+b}{2} = \frac{3+1}{2} = 2$$

$$3) a + m = 140 \quad (1)$$

$$\frac{30}{100} a = \frac{2}{5} m \Rightarrow \frac{3}{10} a = \frac{2}{5} m$$

$$\Rightarrow a = \frac{2}{5} \cdot \frac{10}{3} m \Rightarrow a = \frac{4}{3} m$$

$$\text{înlocuim în (1)} \Rightarrow \frac{4}{3} m + m = 140$$

$$\frac{7}{3} m = 140 \Rightarrow m = 140 \cdot \frac{3}{7} = 60$$

$$m = 60 \text{ și } a = \frac{4}{3} m \Rightarrow a = \frac{4}{3} \cdot 60$$

$\Rightarrow a = 80 \rightarrow$  dăna a economisit  
80 de lei

$$\text{II 4) a) } x = 3^{47} : 3^{45} - 2^{40} : 2^{38}, \text{ se cere } x = 5$$

$$x = 3^{47-45} - 2^{40-38}$$

$$x = 3^2 - 2^2 \Rightarrow x = 9 - 4 \Rightarrow x = 5$$

$$\text{b) } y = \left( \frac{1}{\sqrt{5}} + \sqrt{5} \right) \cdot \sqrt{5} + \left( \sqrt{3} - \frac{1}{\sqrt{3}} \right) : \frac{1}{3\sqrt{3}}$$

se cere cel mai mic număr natural de 2 cifre cu care este divizibil  $N = y - \frac{x+1}{2}$

$$y = \frac{1}{\sqrt{5}} \cdot \sqrt{5} + \sqrt{5} \cdot \sqrt{5} + \left( \sqrt{3} - \frac{1}{\sqrt{3}} \right) \cdot 3\sqrt{3}$$

$$y = 1 + 5 + \sqrt{3} \cdot 3\sqrt{3} - \frac{1}{\sqrt{3}} \cdot 3\sqrt{3}$$

$$y = 6 + 9 - 3 \Rightarrow y = 12$$

$$N = y - \frac{x+1}{2} = 12 - \frac{5+1}{2} = 12 - 3 = 9$$

numerele naturale divizibile cu 9 sunt

$$M_9 = \{ 0; 9; 18; 27; \dots \}$$
 și cel mai mic

număr natural de 2 cifre cerut este 18

$$\text{II 5) } E(x) = (2x+1)^2 - 3(x-1)^2 - (x-1)(x+1) - 6(x+1)$$

$$E(x) = 4x^2 + 4x + 1 - 3(x^2 - 2x + 1) - (x^2 - 1) - 6x - 6$$

$$E(x) = 4x^2 + 4x + 1 - 3x^2 + 6x - 3 - x^2 + 1 - 6x - 6$$

$$E(x) = 4x - 7$$

$$E(n) \leq -1 \Rightarrow 4n - 7 \leq -1 \Rightarrow 4n \leq 6 \quad | :4$$

$$\Rightarrow n \leq \frac{3}{2} \Rightarrow n \in (-\infty; \frac{3}{2}]$$

Se cer numerele naturale din acest interval  $\Rightarrow n \in \{0, 1\}$

III) 1) ABCD paralel.

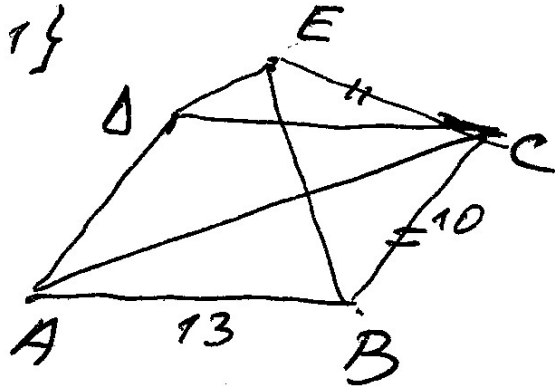
$$AB = 13 \text{ cm}$$

$$BC = 10 \text{ cm}$$

$$m(\widehat{BAA}) < 90^\circ$$

$$DE \parallel AC, DE \subset AC$$

$$BC \equiv CE$$



$$\begin{aligned} a) P_{ABCD} &= 2(AB + BC) = 2(13 \text{ cm} + 10 \text{ cm}) \\ &= 2 \cdot 23 \text{ cm} = 46 \text{ cm} \end{aligned}$$

b) se cere  $AB \equiv AE$  (?)

$$\left. \begin{array}{l} \text{din ip, } BC \equiv CE \\ \text{Dar, } BC \equiv AD \text{ (ABCD paralel)} \end{array} \right\} \Rightarrow AD \equiv CE \quad (1)$$

$$\left. \begin{array}{l} \text{Cum } DE \parallel AC \text{ (ip)} \Rightarrow ACES \text{ trapez} \\ AD \equiv CE \text{ (1)} \end{array} \right\} \Rightarrow$$

$$\left. \begin{array}{l} ACES \text{ trapez isosul} \Rightarrow AE \equiv CD \text{ (diag congr)} \\ \text{Dar, } CD \equiv AB \text{ (ABCD paralel)} \end{array} \right\} \Rightarrow$$

$$\Rightarrow AE \equiv AB$$

III 1c) Dacă  $m(\widehat{BCE}) = 60^\circ$ , arătați că  
aria ABCE este  $60 + 25\sqrt{3} \text{ cm}^2$

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$$A_{ABCE} = A_{BCE} + A_{ABE}$$

$\Delta BCE$  isoscel ( $BC \equiv EC$ )  
 $m(\widehat{BCE}) = 60^\circ$  }  $\Rightarrow \Delta BCE$   
echilateral

$$\Rightarrow A_{BCE} = \frac{l^2 \cdot \sqrt{3}}{4} = \frac{10^2 \sqrt{3}}{4} \text{ cm}^2 = \frac{100\sqrt{3}}{4} \text{ cm}^2 \Rightarrow$$

$$A_{BCE} = 25\sqrt{3} \text{ cm}^2$$

În  $\Delta ABE$ ,  $AB = 13 \text{ cm}$ ,  $AE = 13 \text{ cm}$ ,  $BE = 10 \text{ cm}$

$$\Rightarrow A_{ABE} = \sqrt{p(p-a)(p-b)(p-c)} \text{ unde}$$

$$p = \frac{a+b+c}{2} \text{ (HERON)}$$

$$\text{În cazul nostru, } p = \frac{13+13+10}{2} \text{ cm} = 18 \text{ cm}$$

$$\Rightarrow A_{ABE} = \sqrt{18(18-13)(18-13)(18-10)} \text{ cm}^2$$

$$A_{ABE} = \sqrt{18 \cdot 5 \cdot 5 \cdot 8} \text{ cm}^2$$

$$A_{ABE} = \sqrt{2 \cdot 3^2 \cdot 5^2 \cdot 2^3} \text{ cm}^2 = 4 \cdot 3 \cdot 5 \text{ cm}^2 = 60 \text{ cm}^2$$

$$A_{ABCE} = A_{ABE} + A_{BCE} = 60 + 25\sqrt{3} \text{ cm}^2 \quad \square$$

III 2) ABCD dreptunghi

$$AB = 24 \text{ cm}$$

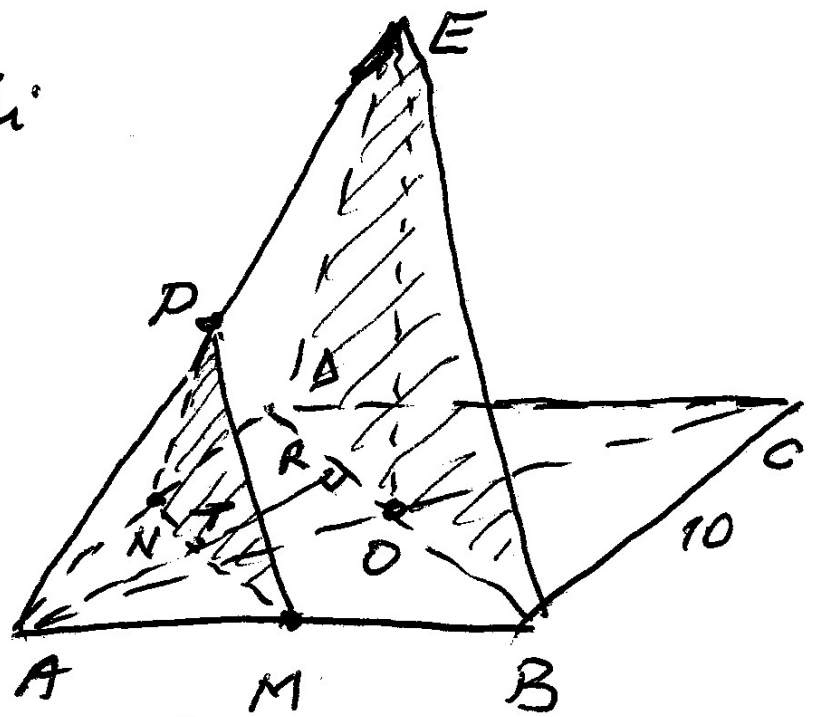
$$BC = 10 \text{ cm}$$

$$AC \cap BD = \{O\}$$

$$EO \perp (ABC)$$

M, N, P mijlocuri

de AB, AD, AE



a) de unde  $S_{ABCD} = 240 \text{ cm}^2$

$$S_{ABCD} = L \cdot l = 24 \text{ cm} \cdot 10 \text{ cm} = 240 \text{ cm}^2$$

b) de unde  $(MNP) \parallel (BDE)$

în  $\triangle ABE$ ,  $[MP]$  linie mijlocie  $\Rightarrow MP \parallel BE$

în  $\triangle ADE$ ,  $[NP]$  linie mijlocie  $\Rightarrow NP \parallel DE$

Asadar,  $MP \parallel BE$

$NP \parallel DE$

$$MP \cap NP = \{P\}$$

$$BE \cap DE = \{E\}$$

$MP, NP \subset (MNP)$

$BE, DE \subset (BDE)$

}  $\Rightarrow (MNP) \parallel (BDE)$

$$\text{III } 2 \text{ c) } \text{se cere dist}((MNP), (BEA)) = \frac{60}{13} \text{ cm}$$

Linie  $AR \perp DB$

$$AR \cap NM = \{T\}$$

$$\text{Avem } \left. \begin{array}{l} EO \perp (ABC) \\ AR \subset (ABC) \end{array} \right\} \Rightarrow EO \perp AR \Rightarrow AR \perp EO$$

Cum  $AR \perp BD$

$$EO \cap DB \neq \emptyset$$

$$\Rightarrow AR \perp (DEB)$$

$$\left. \begin{array}{l} TR \perp (DEB) \\ TE(MNP) \end{array} \right\} \Rightarrow d((MNP), (DEB)) = TR$$

(deoarece distanța dintre 2 plane paralele este distanța de la un punct al unui plan la celălalt plan)

$$TR = \frac{AR}{2}$$

$$\text{În } \triangle DAB \text{ cu } m(\widehat{A}) = 90^\circ \Rightarrow AR = \frac{AD \cdot AB}{DB}$$

$$DB = \sqrt{AD^2 + AB^2} = \sqrt{10^2 + 24^2} = 26$$

$$AR = \frac{24 \cdot 10}{26} = \frac{12 \cdot 10}{13} = \frac{120}{13}$$

$$\Rightarrow TR = \frac{120}{13} \cdot \frac{1}{2} = \frac{60}{13}$$